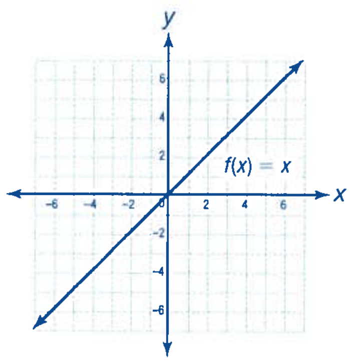
LESSON 18

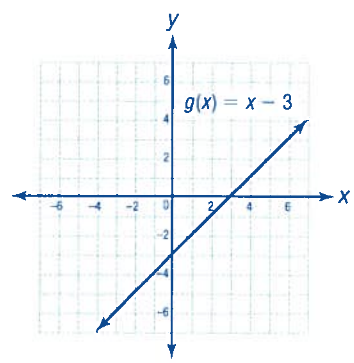
Translating Functions

UNDERSTAND You can think of functions as being grouped into families. All functions in a family have similar characteristics. For example, the graphs of all functions in the family of linear functions are straight lines.

Each family of functions has a **parent function**, the most basic function in the family. The family of linear functions has the parent function $f(x) = x$. The function $f(x) = e^x$ is the general parent function for all exponential functions. However, it can often be easier to group the exponential functions into smaller subfamilies that have the same base, such as $f(x) = 2^x$ and $f(x) = 23.5^x$.

If you change the parent function by adding, subtracting, multiplying, or dividing by a constant, you transform the function and make a new function from the same family. For example, the function $g(x) = x - 3$ is different from the parent function $f(x)= x$, but it is still in the linear function family. Changing the equation of the function also changes the graph of the function. This change to the graph is called a **transformation**.



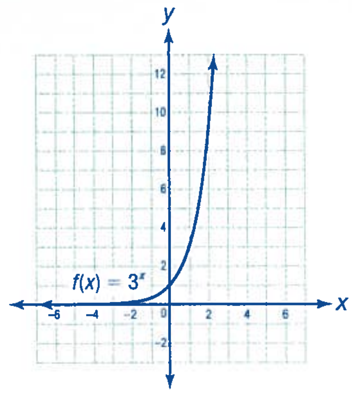


Adding to or subtracting from a function moves its graph up, down, left, or right on the coordinate plane. This kind of transformation is called a **translation**.

Translation	Algebraic Notation	Change to Graph
In a **vertical translation**, every point on the graph shifts up or down.	$g(x) = f(x) + k$ A real number, k, is added to the output, $f(x)$.	If $k > 0$, shift the graph $\|k\|$ units up. If $k < 0$, shift the graph $\|k\|$ units down.
In a **horizontal translation,** every point on the graph shifts left or right.	$g(x) = f(x + k)$ A real number, k, is added to the input, x.	If $k < 0$, shift the graph $\|k\|$ units right. If $k > 0$, shift the graph $\|k\|$ units left.

Connect

The exponential function $f(x) = 3^x$ is graphed on the coordinate plane below. Make a table of values for the function $g(x) = 3^x + 2$. Then graph function g on the same coordinate plane. Describe how function f could be translated to form function g and how translating a function affects its size and shape.



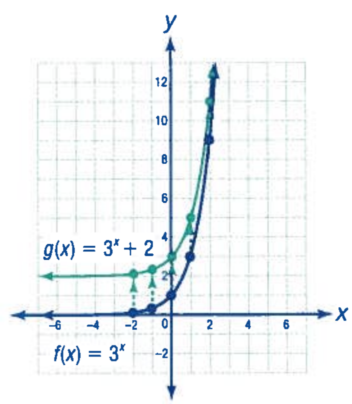
1

Create a table of values for $g(x) = 3^x + 2$.

x	$g(x) = 3^x + 2$	$g(x)$
-2	$g(-2) = 3^{-2} + 2 = \frac{1}{9} + 2 = \frac{19}{9}$	$\frac{19}{9}$
-1	$g(-1) = 3^{-1} + 2 = \frac{1}{3} + 2 = \frac{7}{3}$	$\frac{7}{3}$
0	$g(0) = 3^0 + 2 = 1 + 2 = 3$	3
1	$g(1) = 3^1 + 2 = 3 + 2 = 5$	5
2	$g(2) = 3^2 + 2 = 9 + 2 = 11$	11

2

Plot the ordered pairs for function g and connect them with a curve.



3

Compare the graphs.

- Each point on the graph of function g is 2 units above its corresponding point on function f. So, function g is the result of a vertical translation of function f 2 units up.

Since all we are doing is sliding the graph in the coordinate plane, the size and shape of the graph have not changed.

DISCUSS

Since you were given the graph of $f(x) = 3^x$, could you have graphed $g(x) = 3^x + 2$ without creating a table of values first? Explain.

EXAMPLE A Let $f(x) = 2x$ and define a function g such that $g(x) = f(x + 3)$. Graph both functions, f and g, on the same coordinate plane. Compare the two graphs and identify how function f could be translated to form function g.

1 Write function g in terms of x by using functional notation.

For the function g, use the expression for $f(x)$ and replace x with $(x + 3)$.

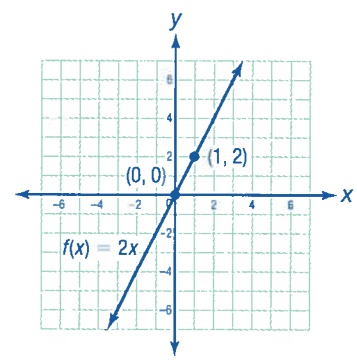
$g(x) = f(x + 3)$

$g(x) = 2(x + 3)$

$g(x) = 2x + 6$

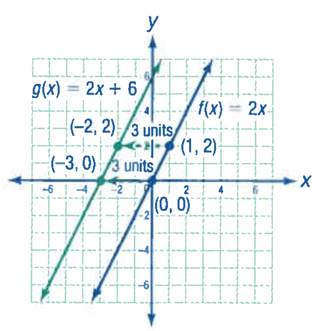
2 Graph function f.

The graph of f is the graph of the equation $y = f(x)$ or $y = 2x$. This equation has a slope of 2 and a y-intercept at (0, 0). Graph the y-intercept, use the slope to find another point, and draw a straight line through those points.



3 Graph function g.

The graph of g is the graph of the equation $y = g(x)$ or $y = 2x + 6$. This equation has a slope of 2 and a y-intercept at (0, 6). Graph the y-intercept, use the slope to find another point, and draw a straight line through those points.



4 Compare the graphs.

- Each point on function g is 3 units to the left of the corresponding point on function f.

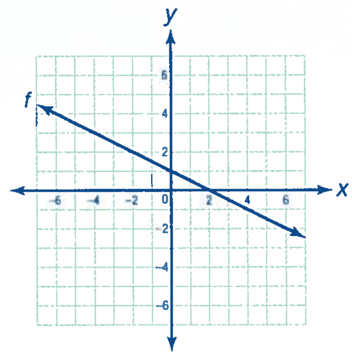
This makes sense. When 3 is added to the input, as it was in $g(x) = f(x + 3)$, the result is a translation of 3 units to the left.

Notice that both lines have the same slope, 2. So, translating a line horizontally does not change its slope.

TRY

On the grid shown in Step 3 above, graph $h(x) = f(x - 3)$. Describe how function f could be translated to form function h.

EXAMPLE B A linear function f is graphed below. On the same coordinate plane, graph the function $g(x) = f(x) - 5$. Identify the transformation.



1 Describe the translation.

$g(x) = f(x) - 5$ is in the form $g(x) = f(x) + k$, where $k = -5$.

When a numerical value, k, is added to an output, $f(x)$, the result is a vertical shift.

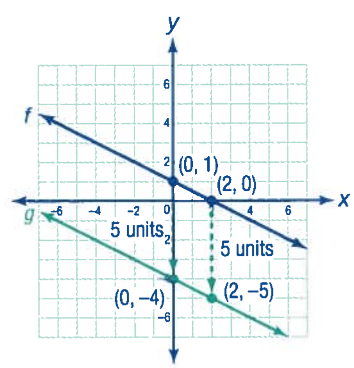
Since k is the negative number -5, shift the graph 5 units down.

2 Graph the function g.

Shift two points on the graph of function f down 5 units. Then draw a line through them.

(0, 1) is translated 5 units down to (0, −4).

(2, 0) is translated 5 units down to (2, −5).

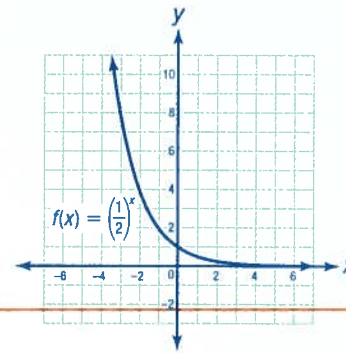


The transformation is a vertical translation 5 units down.

DISCUSS

The equations for functions f and g are not given. Can you determine if their slopes are the same? Explain how.

EXAMPLE C The graph of $f(x) = \left(\frac{1}{2}\right)^x$ is shown. Translate function f to form the function $h(x) = f(x - 4) + 2$. Graph h and write its explicit equation.



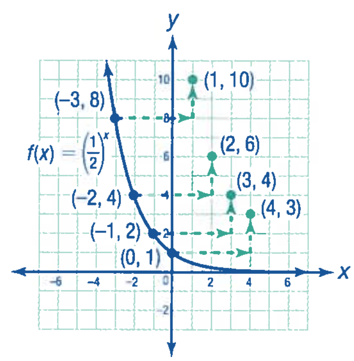
1 Describe the translation by using words and symbols.

Subtracting 4 from the input, x, indicates a horizontal translation. Translate the graph 4 units to the right.

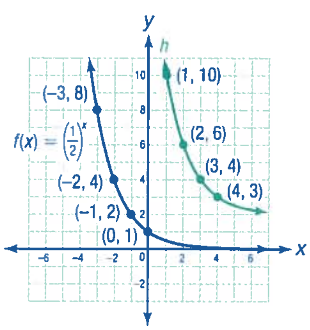
Adding 2 to the output, $f(x - 4)$, indicates a vertical translation. Translate the graph 2 units up.

2 Graph h.

Choose several points on the graph of f and translate each point 4 units to the right and 2 units up.



3 Connect the points with a smooth curve.



4 Write an equation for h.

The translation involved subtracting 4 from the input and adding 2 to that output. So, subtract 4 from x, which is the exponent in $\left(\frac{1}{2}\right)^x$, and then add 2 to the resulting expression.

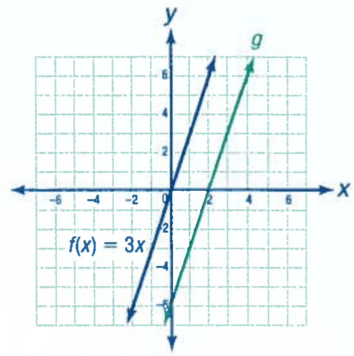
▶ $h(x) = \left(\frac{1}{2}\right)^{x-4} + 2.$

CHECK

Use a graphing calculator to check your work. Press Y=.

For Y_1, enter (1/2)^X. For Y_2, enter (1/2)^(X – 4) + 2. Press 2nd GRAPH to bring up a table of values. Press GRAPH to view the graph.

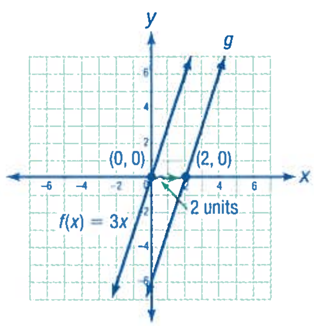
EXAMPLE D Functions f and g are graphed on the right. Using function notation, write an equation describing $g(x)$ in terms of $f(x)$. Then use the equation given for $f(x)$ to write an equation for $g(x)$ in terms of x.



1 Identify how f could be translated to form g.

Choose a point on f, such as (0, 0).

If this point is translated 2 units to the right, it would cover point (2, 0), which is on the graph of g.



Verify that any point on f, if translated 2 units to the right, has a corresponding point on g.

2 Write the function $g(x)$ in terms of $f(x)$.

To represent a horizontal translation of 2 units to the right, add -2 to the input, x.

So $g(x) = f(x - 2)$.

3 Write the function $g(x)$ in terms of x.

To find an explicit expression for $g(x)$, substitute $(x - 2)$ for x in the expression for $f(x)$.

$g(x) = f(x - 2)$

$g(x) = 3(x - 2)$

▶ $g(x) = 3x - 6$

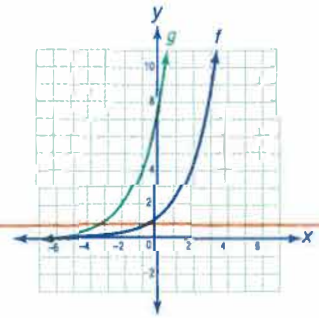
CHECK

Use a graphing calculator to check that $g(x) = 3x - 6$ is the correct equation for function g.

Practice

Use words to describe how function *f* could be translated to form function *g* in one step.

1.



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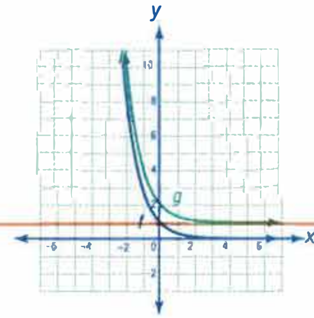

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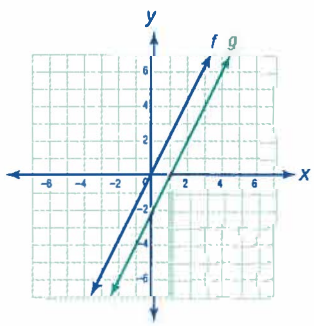
HINT How can (0, 1) be translated to cover (0, 2)?

Use words to describe a horizontal translation that would transform function *f* into function *g*. Then describe a vertical translation that would transform function *f* into function *g*.

3.

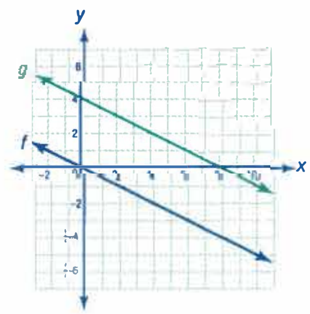


horizontal translation: ______________

vertical translation: ______________

REMEMBER Horizontal means left and right. Vertical means up and down.

4.



horizontal translation: ______________

vertical translation: ______________

Write *true* or *false* for each statement. If false, rewrite the statement to make it true.

5. A translation is a slide of a graph to a new location on the coordinate plane.

____

6. If $g(x) = f(x) - k$, then the graph of f is translated k units down to form the graph of g.

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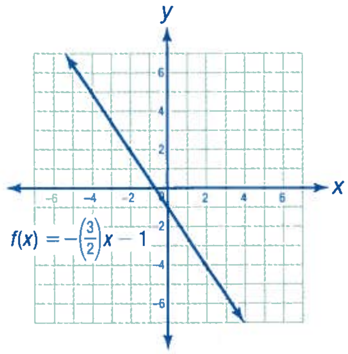
7. If $g(x) = f(x - k)$, then the graph of f is translated k units left to form the graph of g.

____

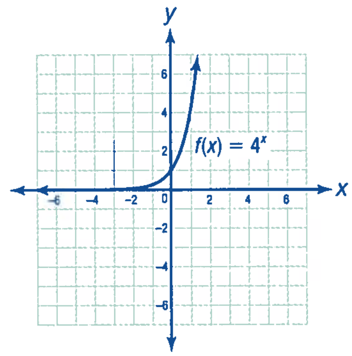
Translate the graph of *f* according to the verbal description to form *g* and draw the graph for *g* on the same coordinate plane. Then write an equation for $g(x)$ in terms of *x*.

8. Translate the graph of *f* 5 units up to form *g*.



$g(x) =$ ______________

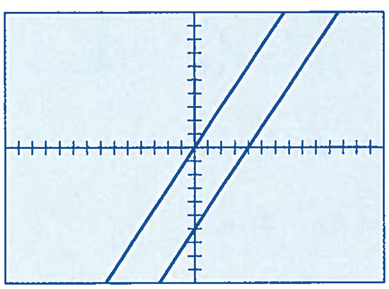
9. Translate the graph of *f* 3 units to the right to form *g*.



$g(x) =$ ______________

Choose the best answer. Use your graphing calculator to check your answer.

10. The graphing calculator screen below shows the graph of $f(x) = 1.5x$ and the graph of *g*.



Which equation could represent $g(x)$ in terms of $f(x)$?

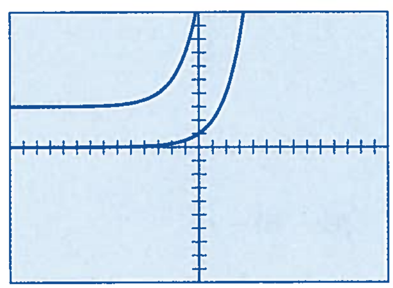
A. $g(x) = f(x) + 6$

B. $g(x) = f(x) - 6$

C. $g(x) = f(x + 6)$

D. $g(x) = 6f(x)$

11. The graphing calculator screen below shows the graph of $f(x) = 2^x$ and the graph of *g*.



Which equation could represent $g(x)$ in terms of $f(x)$?

A. $g(x) = f(x + 3) + 3$

B. $g(x) = f(x - 3) + 3$

C. $g(x) = f(x + 3) - 3$

D. $g(x) = f(x - 3) - 3$

Write an explicit expression in terms of x for each function $g(x)$ described below. For questions 12–17, $f(x) = 5^x$.

12. translation of $f(x)$ 2 units up

$g(x) =$ ________________

13. translation of $f(x)$ 2 units down

$g(x) =$ ________________

14. translation of $f(x)$ 2 units left

$g(x) =$ ________________

15. translation of $f(x)$ 2 units right

$g(x) =$ ________________

16. translation of $f(x)$ 3 units right and 3 units up

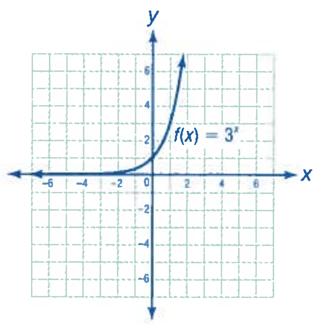
$g(x) =$ ________________

17. translation of $f(x)$ 3 units left and 3 units down

$g(x) =$ ________________

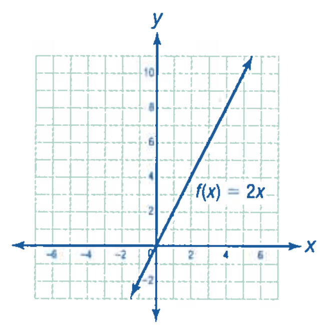
Translate the graph of function f to form the translated function g described algebraically. Write an equation in terms of x to represent the translated image.

18. $g(x) = f(x) + 5$



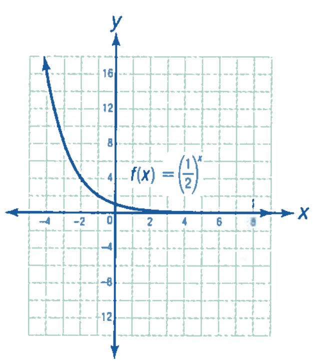
$g(x) =$ ________________

19. $g(x) = f(x + 5)$



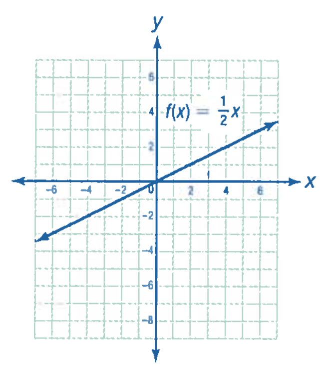
$g(x) =$ ________________

20. $g(x) = f(x - 6) - 4$



$g(x) =$ ________________

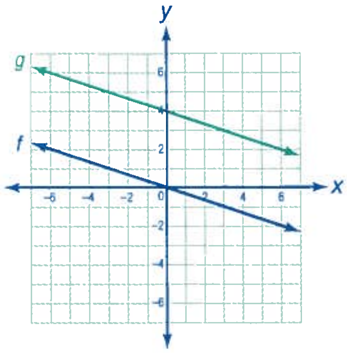
21. $g(x) = f(x - 4) - 3$



$g(x) =$ ________________

Function *f* was translated to form function *g* according to the rule given. For each rule, identify the value of *k*. Include the sign. Briefly explain how you know.

22.



$g(x) = f(x) + k$; $k =$ ______

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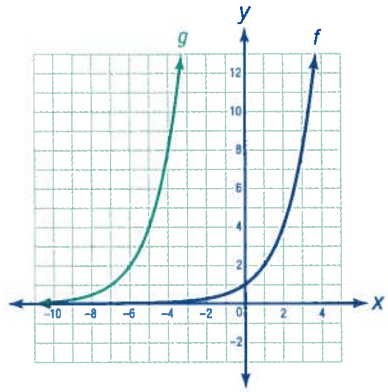
23.



$g(x) = f(x + k)$; $k =$ ______

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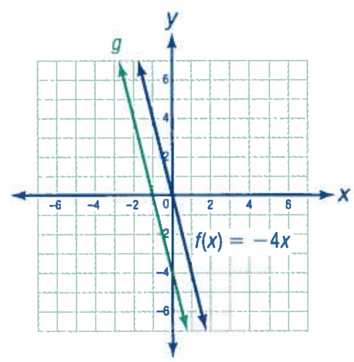
Examine the following situations and respond in complete sentences.

24. **EXPLAIN** Macy graphed $f(x) = -4x$ and parallel line g. She believes that since the rule for the translation is $g(x) = f(x + 1)$, the equation for g must be $g(x) = -4x + 1$. Explain why Macy's reasoning is flawed. Then identify the correct equation for g.



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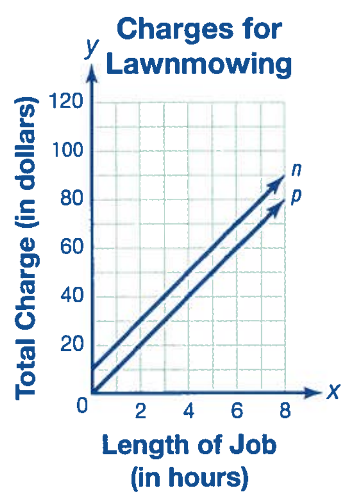
25. **DESCRIBE** Zack used to charge only an hourly rate to mow lawns, as shown by the graph of function p. Because of rising costs, he now charges a set fee for each job in addition to his hourly rate, as shown by the graph of function n. Use algebraic notation to describe how p could be translated to form n. Use what you know about translations to explain how the new costs differ from the old costs.



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