

8-1 Adding and Subtracting Polynomials

A.SSE.1a, A.APR.1

Write each polynomial in standard form. **14.** $3x^2 + x + 2$

14. $x + 2 + 3x^2$ **15.** $1 - x^4 - x^4 + 1$

16. $2 + 3x + x^2$ **17.** $3x^5 - 2 + 6x - 2x^2 + x^3$
 $x^2 + 3x + 2$ $3x^5 + x^3 - 2x^2 + 6x - 2$

Find each sum or difference.

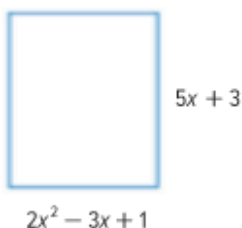
18. $(x^3 + 2) + (-3x^3 - 5)$ $-2x^3 - 3$

19. $a^2 + 5a - 3 - (2a^2 - 4a + 3)$ $-a^2 + 9a - 6$

20. $(4x - 3x^2 + 5) + (2x^2 - 5x + 1)$ $-x^2 - x + 6$

- 21. PICTURE FRAMES** Jean is framing a painting that is a rectangle. What is the perimeter of the frame?

$4x^2 + 4x + 8$



Example 1

Write $3 - x^2 + 4x$ in standard form.

Step 1 Find the degree of each term.

3 : degree 0

$-x^2$: degree 2

$4x$: degree 1

Step 2 Write the terms in descending order of degree.

$3 - x^2 + 4x = -x^2 + 4x + 3$

Example 2

Find $(8r^2 + 3r) - (10r^2 - 5)$.

$(8r^2 + 3r) - (10r^2 - 5)$

$= (8r^2 + 3r) + (-10r^2 + 5)$

$= (8r^2 - 10r^2) + 3r + 5$

$= -2r^2 + 3r + 5$

Use the additive inverse.

Group like terms.

Add like terms.

8-2 Multiplying a Polynomial by a Monomial

A.APR.1

Solve each equation.

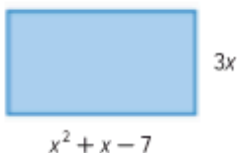
22. $x^2(x + 2) = x(x^2 + 2x + 1)$ **0**

23. $2x(x + 3) = 2(x^2 + 3)$ **1**

24. $2(4w + w^2) - 6 = 2w(w - 4) + 10$ **1**

- 25. GEOMETRY** Find the area of the rectangle.

$3x^3 + 3x^2 - 21x$



Example 3

Solve $m(2m - 5) + m = 2m(m - 6) + 16$.

$m(2m - 5) + m = 2m(m - 6) + 16$

$2m^2 - 5m + m = 2m^2 - 12m + 16$

$2m^2 - 4m = 2m^2 - 12m + 16$

$-4m = -12m + 16$

$8m = 16$

$m = 2$

8-3 Multiplying Polynomials

A.APR.1

Find each product. **26.** $x^2 + 4x - 21$ **27.** $18a^2 + 3a - 10$

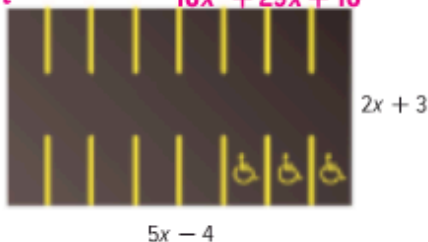
26. $(x - 3)(x + 7)$ **27.** $(3a - 2)(6a + 5)$

28. $(3r - 7t)(2r + 5t)$ **29.** $(2x + 5)(5x + 2)$
 $6r^2 + rt - 35t^2$ $10x^2 + 29x + 10$

- 30. PARKING LOT**

The parking lot shown is to be paved. What is the area to be paved?

$10x^2 + 7x - 12$



Example 4

Find $(6x - 5)(x + 4)$.

$(6x - 5)(x + 4)$

F **O** **I** **L**

$= (6x)(x) + (6x)(4) + (-5)(x) + (-5)(4)$

$= 6x^2 + 24x - 5x - 20$ Multiply.

$= 6x^2 + 19x - 20$ Combine like terms.

8-4 Special Products

A.APR.1

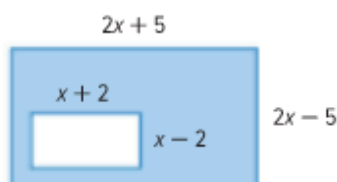
Find each product.

31. $(x + 5)(x - 5)$ $x^2 - 25$ 32. $(3x - 2)^2$ $9x^2 - 12x + 4$

33. $(5x + 4)^2$ $25x^2 + 40x + 16$ 34. $(2x - 3)(2x + 3)$ $4x^2 - 9$

35. $(2r + 5t)^2$ $4r^2 + 20rt + 25t^2$ 36. $(3m - 2)(3m + 2)$

37. **GEOMETRY** Write an expression to represent the area of the shaded region. $3x^2 - 21$



Example 5

Find $(x - 7)^2$.

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$\begin{aligned}(x - 7)^2 &= x^2 - 2(x)(7) + (-7)^2 \\ &= x^2 - 14x + 49\end{aligned}$$

Square of a Difference

$$a = x \text{ and } b = 7$$

Simplify.

Example 6

Find $(5a - 4)(5a + 4)$.

$$(a + b)(a - b) = a^2 - b^2$$

$$\begin{aligned}(5a - 4)(5a + 4) &= (5a)^2 - (4)^2 \\ &= 25a^2 - 16\end{aligned}$$

Product of a Sum and Difference

$$a = 5a \text{ and } b = 4$$

Simplify.

8-5 Using the Distributive Property

A.SSE.2, A.SSE.3a

Use the Distributive Property to factor each polynomial.

38. $12x + 24y$ $12(x + 2y)$

39. $14x^2y - 21xy + 35xy^2$ $7xy(2x - 3 + 5y)$

40. $8xy - 16x^3y + 10y$ $2y(4x - 8x^3 + 5)$

41. $a^2 - 4ac + ab - 4bc$ $(a + b)(a - 4c)$

42. $2x^2 - 3xz - 2xy + 3yz$ $(2x - 3z)(x - y)$

43. $24am - 9an + 40bm - 15bn$ $(3a + 5b)(8m - 3n)$

Factor each expression.

44. $24ab + 54a - 20b - 45$ $(6a - 5)(4b + 9)$

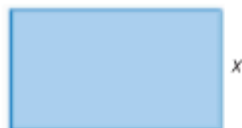
45. $3r^3 - 12r^2 + 4p - pr$ $(3r^2 + p)(r - 4)$

46. $6c^2d + 30cd$ $6cd(c + 5)$

47. $18f^4g^5 - 3f^6g^2 + 9f^4g^3$ $3f^4g^2(6g^3 - f^2 + 3g)$

48. **GEOMETRY** The area of the rectangle shown is $x^3 - 2x^2 + 5x$ square units. What is the length?

$$x^2 - 2x + 5$$



Example 7

Factor $12y^2 + 9y + 8y + 6$.

$$12y^2 + 9y + 8y + 6$$

$$= (12y^2 + 9y) + (8y + 6)$$

$$= 3y(4y + 3) + 2(4y + 3)$$

$$= (4y + 3)(3y + 2)$$

Group terms with common factors.

Factor the GCF from each group.

Distributive Property

Example 8

Factor $5a^2 - 10ab + 6b - 3a$.

$$5a^2 - 10ab + 6b - 3a$$

$$= (5a^2 - 10ab) + (6b - 3a)$$

$$= 5a(a - 2b) + 3(2b - a)$$

$$= 5a(a - 2b) + 3[-1](a - 2b)$$

$$= 5a(a - 2b) - 3(a - 2b)$$

$$= (5a - 3)(a - 2b)$$

Group terms with common factors.

Factor the GCF from each group.

$$2b - a = -1(a - 2b)$$

Associative Property

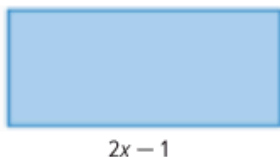
Distributive Property

Factor each polynomial.

49. $x^2 - 8x + 15$ $(x - 5)(x - 3)$
 50. $x^2 + 9x + 20$ $(x + 5)(x + 4)$
 51. $x^2 - 5x - 6$ $(x - 6)(x + 1)$
 52. $x^2 + 3x - 18$ $(x + 6)(x - 3)$
 53. $x^2 + 5x - 50$ $(x + 10)(x - 5)$
 54. $x^2 - 6x + 8$ $(x - 2)(x - 4)$
 55. $x^2 + 12x + 32$ $(x + 4)(x + 8)$
 56. $x^2 - 2x - 48$ $(x + 6)(x - 8)$
 57. $x^2 + 11x + 10$ $(x + 10)(x + 1)$
58. **ART** An artist is working on a painting. If the area of the canvas is represented by $x^2 + 2x - 24$, what are the dimensions of the canvas? $(x + 4)$ and $(x - 6)$

Factor each trinomial, if possible. If the trinomial cannot be factored, write *prime*.

59. $12x^2 + 22x - 14$ $2(2x - 1)(3x + 7)$
 60. $2y^2 - 9y + 3$ **prime**
 61. $3x^2 - 6x - 45$ $3(x - 5)(x + 3)$
 62. $2a^2 + 13a - 24$ $(2a - 3)(a + 8)$
 63. $20x^2 + x - 12$ $(4x - 3)(5x + 4)$
 64. $2x^2 - 3x - 20$ $(x - 4)(2x + 5)$
 65. $3x^2 - 13x - 10$ $(3x + 2)(x - 5)$
 66. $6x^2 - 7x - 5$ $(3x + 5)(2x + 1)$
67. **GEOMETRY** The area of the rectangle shown is $6x^2 + 11x - 7$ square units. What is the width of the rectangle? $3x + 7$

**Example 9**

Factor $x^2 + 10x + 21$.

$b = 10$ and $c = 21$, so $m + p$ is positive and mp is positive. Therefore, m and p must both be positive. List the positive factors of 21, and look for the pair of factors with a sum of 10.

Factors of 21	Sum of 10
1, 21	22
3, 7	10

The correct factors are 3 and 7.

$$\begin{aligned} x^2 + 10x + 21 &= (x + m)(x + p) && \text{Write the pattern.} \\ &= (x + 3)(x + 7) && m = 3 \text{ and } p = 7 \end{aligned}$$

Example 10

Factor $12a^2 + 17a + 6$.

$a = 12$, $b = 17$, and $c = 6$. Since b is positive, $m + p$ is positive. Since c is positive, mp is positive. So, m and p are both positive. List the factors of $12(6)$ or 72, where both factors are positive.

Factors of 72	Sum of 17
1, 72	73
2, 36	38
3, 24	27
4, 18	22
6, 12	18
8, 9	17

The correct factors are 8 and 9.

$$\begin{aligned} 12a^2 + 17a + 6 &= 12a^2 + ma + pa + 6 \\ &= 12a^2 + 8a + 9a + 6 \\ &= (12a^2 + 8a) + (9a + 6) \\ &= 4a(3a + 2) + 3(3a + 2) \\ &= (3a + 2)(4a + 3) \end{aligned}$$

So, $12a^2 + 17a + 6 = (3a + 2)(4a + 3)$.

Factor each polynomial.

68. $y^2 - 81$ $(y + 9)(y - 9)$

69. $64 - 25x^2$ $(8 + 5x)(8 - 5x)$

70. $16a^2 - 21b^2$ **prime**

71. $3x^2 - 3$ $3(x + 1)(x - 1)$

72. $a^2 - 25$ $(a + 5)(a - 5)$

73. $9x^2 - 25$ $(3x - 5)(3x + 5)$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

74. $x^2 + 12x + 36$ $(x + 6)^2$

75. $x^2 + 5x + 25$ **prime**

76. $9y^2 - 12y + 4$ $(3y - 2)^2$

77. $4 - 28a + 49a^2$ $(2 - 7a)^2$

78. $x^4 - 1$ $(x^2 + 1)(x + 1)(x - 1)$

79. $x^4 - 16x^2$ $x^2(x + 4)(x - 4)$

80. $9x^2 + 25$ **prime**

81. $-3x^2 - 12x - 12$ $-3(x + 2)^2$

Example 11

Factor $x^4 - 16$.

$$x^4 - 16$$

$$= (x^2)^2 - 4^2$$

$$= (x^2 - 4)(x^2 + 4)$$

$$= (x + 2)(x - 2)(x^2 + 4)$$

Original expression

Difference of squares

Factor the difference of squares.

Factor the second difference of squares.

Example 12

Factor $4x^2 + 16x + 16$.

$$4x^2 + 16x + 16 = (2x)^2 + 2(2x)(4) + 4^2$$

$$= (2x + 4)^2$$

Write as $a^2 + 2ab + b^2$.

Factor using the pattern.

