9-1 Graphing Quadratic Functions

F.IF.4, F.IF.7a

Consider each equation.

a. Determine whether the function has a *maximum* or *minimum* value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

11. $y = x^2 - 4x + 4$ **11–14. See margin.**

12. $y = -x^2 + 3x$

13. $y = x^2 - 2x - 3$

14. $y = -x^2 + 2$.

15. **ROCKET** A toy rocket is launched with an upward velocity of 32 feet per second. The equation $h = -16t^2 + 32t$ gives the height of the ball t seconds after it is launched.

    a. Determine whether the function has a *maximum* or *minimum* value. **maximum**

    b. State the maximum or minimum value. **16**

    c. State a reasonable domain and range for this situation.
    $D = \{t \mid 0 \le t \le 2\}$; $R = \{h \mid 0 \le h \le 16\}$

Example 1

Consider $f(x) = x^2 + 6x + 5$.

a. Determine whether the function has a *maximum* or *minimum* value.

For $f(x) = x^2 + 6x + 5$, $a = 1$, $b = 6$, and $c = 5$.

Because a is positive, the graph opens up, so the function has a minimum value.

b. State the *maximum* or *minimum* value of the function.

The minimum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $\frac{-b}{2a}$ or $\frac{-6}{2(1)}$ or -3.

$f(x) = x^2 + 6x + 5$ — Original function

$f(-3) = (-3)^2 + 6(-3) + 5$ — $x = -3$

$f(-3) = -4$ — Simplify.

The minimum value is -4.

c. State the domain and range of the function.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or $\{y \mid y \ge -4\}$.

11a. minimum

11b. 0

11c. $D = \{-\infty < x < \infty\}$; $R = \{y \mid y \ge 0\}$

12a. maximum

12b. 2.25

12c. $D = \{-\infty < x < \infty\}$; $R = \{y \mid y \le 2.25\}$

13a. minimum

13b. -4

13c. $D = \{-\infty < x < \infty\}$; $R = \{y \mid y \ge -4\}$

14a. maximum

14b. 2

14c. $D = \{-\infty < x < \infty\}$; $R = \{y \mid y \le 2\}$

9-2 Transformations of Quadratic Functions

F.IF.7a, F.BF.3

Describe the transformations in each function as it relates to the graph of $f(x) = x^2$.

16–21. See margin.

16. $f(x) = x^2 + 8$

17. $f(x) = x^2 - 3$

18. $f(x) = 2(x - 1)^2$

19. $f(x) = 4x^2 - 18$

20. $f(x) = \frac{1}{3}x^2$

21. $f(x) = \frac{1}{4}x^2$

Example 2

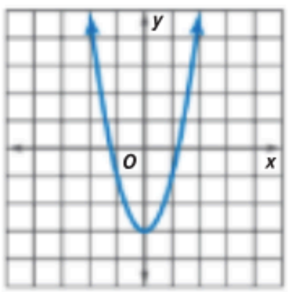
Describe the transformations in $f(x) = x^2 - 2$ as it relates to the graph of $f(x) = x^2$.

The graph of $f(x) = x^2 + k$ represents a translation up or down of the parent graph.

Since $k = -2$, the translation is down.

So, the graph is translated down 2 units from the parent function.

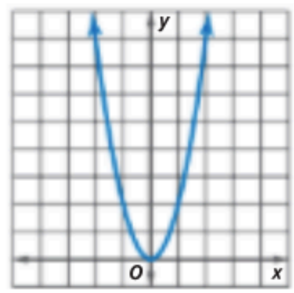
22. Write an equation for the function shown in the graph. **$y = 2x^2 - 3$**



23. **PHYSICS** A ball is dropped off a cliff that is 100 feet high. The function $h = -16t^2 + 100$ models the height h of the ball after t seconds. Compare the graph of this function to the graph of $h = t^2$. **See margin.**

Example 3

Write an equation for the function shown in the graph.



Since the graph opens upward, the leading coefficient must be positive. The parabola has not been translated up or down, so $c = 0$. Since the graph is stretched vertically, it must be of the form of $f(x) = ax^2$ where $a > 1$. The equation for the function is $y = 2x^2$.

16. translated up 8 units

17. translated down 3 units

18. vertical stretch and translated right 1 unit

19. vertical stretch and translated down 18 units

20. vertical compression

21. vertical compression

23. reflected across the x-axis, vertically stretched and translated up 100 units

9-3 Solving Quadratic Equations by Graphing

A.REI.4b, F.IF.7a

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

24. $x^2 - 3x - 4 = 0$ **−1, 4**

25. $-x^2 + 6x - 9 = 0$ **3**

26. $x^2 - x - 12 = 0$ **−3, 4**

27. $x^2 + 4x - 3 = 0$ **−4.6, 0.6**

28. $x^2 - 10x = -21$ **3, 7**

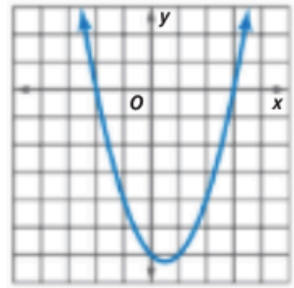
29. $6x^2 - 13x = 15$ **−0.8, 3**

30. **NUMBER THEORY** Find two numbers that have a sum of 2 and a product of −15. **−3 and 5**

Example 4

Solve $x^2 - x - 6 = 0$ by graphing.

Graph the related function $f(x) = x^2 - x - 6$.



The x-intercepts of the graph appear to be at −2 and 3, so the solutions are −2 and 3.

9-4 Solving Quadratic Equations by Factoring

A.SSE.3a, A.REI.4b

Solve each equation. Check your solutions.

31. $x^2 + 6x - 55 = 0$ **−11, 5**

32. $(g + 8)^2 = 49$ **−1, −15**

33. $y^2 - 4y = 32$ **−4, 8**

34. $3k^2 - 8k = 3$ $-\frac{1}{3}, 3$

35. $2n^2 + 4n = 16$ **−4, 2**

36. $4w^2 + 9 = 12w$ $\frac{3}{2}$

Example 5

Solve $x^2 - 2x = 120$ by factoring.

Write the equation in standard form and factor.

$x^2 - 2x = 120$	Original equation
$x^2 - 2x - 120 = 0$	Subtract 120 from each side.
$(x - 12)(x + 10) = 0$	Factor.
$x - 12 = 0$ or $x + 10 = 0$	Zero Product Property
$x = 12$ $\quad x = -10$	Solve each equation

The solutions are 12 and −10.

9-5 Solving Quadratic Equations by Completing the Square

A.SSE.3b, A.REI.4, F.IF.8a

Solve each equation by completing the square. Round to the nearest tenth if necessary.

37. $x^2 + 6x + 9 = 16$ **1, −7**

38. $-a^2 - 10a + 25 = 25$ **0, −10**

39. $y^2 - 8y + 16 = 36$ **10, −2**

40. $y^2 - 6y + 2 = 0$ **5.6, 0.4**

41. $n^2 - 7n = 5$ **−0.7, 7.7**

42. $-3x^2 + 4 = 0$ **−1.2, 1.2**

43. $a^2 - 4a + 9 = 0$ **no solution**

44. $2a^2 - 4a + 1 = 0$ **1.7, 0.3**

45. **NUMBER THEORY** Find two numbers that have a sum of −2 and a product of −48. **−8, 6**

Example 6

Solve $x^2 - 16x + 32 = 0$ by completing the square. Round to the nearest tenth if necessary.

Isolate the x^2- and x-terms. Then complete the square and solve.

$x^2 - 16x + 32 = 0$	Original equation
$x^2 - 16x = -32$	Isolate the x^2- and x-terms.
$x^2 - 16x + 64 = -32 + 64$	Complete the square.
$(x - 8)^2 = 32$	Factor.
$x - 8 = \pm\sqrt{32}$	Take the square root.
$x = 8 \pm \sqrt{32}$	Add 8 to each side.
$x = 8 \pm 4\sqrt{2}$	Simplify.

The solutions are about 2.3 and 13.7.

9-6 Solving Quadratic Equations by Using the Quadratic Formula

A.REI.4b, A.CED.1

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

46. $x^2 - 8x = 20$ **−2, 10**

47. $21x^2 + 5x - 7 = 0$ **−0.7, 0.5**

48. $d^2 - 5d + 6 = 0$ **2, 3**

49. $2f^2 + 7f - 15 = 0$ **−5, 1.5**

50. $2h^2 + 8h + 3 = 3$ **−4, 0**

51. $4x^2 + 4x = 15$ **−2.5, 1.5**

52. **GEOMETRY** The area of a square can be quadrupled by increasing the side length and width by 4 inches. What is the side length? **4 in.**

State the discriminant for each equation. Then determine the number of real solutions of the equation.

53. $a^2 - 4a + 5 = 0$ **−4; no real solutions**

54. $-6x^2 + 2x + 3 = 0$ **76; two real solutions**

Example 7

Solve $x^2 + 10x + 9 = 0$ by using the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-10 \pm \sqrt{10^2 - 4(1)(9)}}{2(1)}$ $a = 1, b = 10, c = 9$

$= \frac{-10 \pm \sqrt{64}}{2}$ Simplify.

$x = \frac{-10 + 8}{2}$ or $x = \frac{-10 - 8}{2}$ Separate the solutions.

$= -1$ $= -9$ Simplify.

9-7 Solving Systems of Linear and Quadratic Equations

A.REI.7, A.CED.2

Solve each system of equations.

55. $y = x^2 - 4x + 4$ **(1, 1), (5, 9)**
$y = 2x - 1$

56. $y = x^2 + 5x - 3$ **(−9, 33), (2, 11)**
$y = 15 - 2x$

57. $y = 2x^2 - 4x + 1$ $\left(\frac{1}{2}, -\frac{1}{2}\right)$ **(4, 17)**
$y - 5x = -3$

58. $y = 4x^2 + 4x - 3$ **(−2, 5),** $\left(\frac{3}{4}, 2\frac{1}{4}\right)$
$x + y = 3$

59. $y = x^2 - 4x + 9$ **(3, 0)**
$y = 2x$

60. $y = x^2 + 9$ **(1, 10), (9, 90)**
$y = 10x$

61. $y = x^2 + 9$ **no solution**
$y = x$

Example 8

Solve the system of equations $\begin{cases} y = x^2 + 2x \\ y = x + 2 \end{cases}$.

The equations are solved for y. Substitute and solve for x.

$x^2 + 2x = x + 2$ Substitute.

$x^2 + x = 2$ Subtract x from each side.

$x^2 + x - 2 = 0$ Subtract 2 from each side.

$(x + 2)(x - 1) = 0$ Factor.

$x + 2 = 0$ or $x - 1 = 0$ Zero Product Property

$x = -2$ or $x = 1$ Solve for x.

Substitute to find the values of y.

$y = x + 2$	$y = x + 2$
$y = (-2) + 2$	$y = 1 + 2$
$y = 0$	$y = 3$

The solutions are $(-2, 0)$ and $(1, 3)$.

9-9 Combining Functions

F.BF.1b

Consider the following functions.

$f(x) = 4x^2 - 8x + 2$

$g(x) = -5x^2 + x + 1$

$h(x) = 3 - x^2$

Determine each of the following.

66. $(f + g)(x)$ $-x^2 - 7x + 3$
67. $(g + h)(x)$ $-6x^2 + x + 4$
68. $(f - h)(x)$ $5x^2 - 8x - 1$
69. $(h - g)(x)$ $4x^2 - x + 2$
70. $(f \cdot h)(x)$ $-4x^4 + 8x^3 + 10x^2 - 24x + 6$
71. $(g \cdot h)(x)$ $5x^4 - x^3 - 16x^2 + 3x + 3$

Example 10

Let $f(x) = -2x^2 - x + 5$ and $g(x) = x^2 - 1$. Determine $(f \cdot g)(x)$ and $(f - g)(x)$.

$(f \cdot g)(x)$	Original expression
$= (-2x^2 - x + 5)(x^2 - 1)$	Substitute.
$= -2x^4 + 2x^2 - x^3 + x + 5x^2 - 5$	Distribute.
$= -2x^4 - x^3 + 7x^2 + x - 5$	Combine like terms.

$(f - g)(x)$	Original expression
$= (-2x^2 - x + 5) - (x^2 - 1)$	Substitute.
$= -2x^2 - x + 5 - x^2 + 1$	Distribute.
$= -3x^2 - x + 6$	Combine like terms.